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Phase Retrieval*

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Abstract. The problem of phase retrieval arises in experimental uses of diffraction to determine intrinsic structure because the modulus of a Fourier transform is all that can usually be measured after diffraction occurs. For finite distributions, the phase retrieval problem can be solved by methods of factorization in suitable rings of polynomials; for continuous distributions with compact support, the methods of complex analysis are needed to solve the phase retrieval problem. These methods are discussed and examples are given for illustration.

1. Introduction

The problem of phase retrieval arises in all experimental uses of diffracted electromagnetic radiation for determining the intrinsic detailed structure of a diffracting object. Usually, the measurement of the diffracted wavefront gives only the approximate values of the intensity of this wave form, and not its complex amplitude. Consequently, the phase information is not known explicitly and must be determined by other methods. Because most diffracted wavefronts in experimental situations are approximated by a Fourier transform or Fourier series in one or several variables, the methods of Fourier analysis are a great help to phase retrieval procedures. Using Fourier transforms, one sees that many phase retrieval problems are algebraic in nature; this algebraic aspect of phase retrieval is one of the main themes presented here.

A point source of monochromatic electromagnetic radiation falling on an opaque screen with slits or punctures will diffract in passing through the openings of the screen and create a new wave front that can be described precisely by Kirchoff's integral representation, which is a general type of integral transform. This Fresnel diffraction simplifies if one makes the assumption that the distance of the source to the screen and of the screen to the focusing plane of diffraction are both large with respect to the size of the holes in the screen and the wave length of

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