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Detailed Balance and Equilibrium

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Abstract. For classical lattice systems, an infinite set of jump-processes satisfying the condition of detailed balance is found. It is proved that any state invariant for these processes is an equilibrium state, providing a new characterization of DLR-states by means of the notion of detailed balance. This extends previous results, proved in one and two dimensions.

I. Introduction

Since their first appearance [1, 2] in rigorous statistical mechanics, the equilibrium conditions, known as the DLR-conditions, have been reformulated in various alternative ways. Without being exhaustive at all, one has proved that the translation invariant DLR-states minimize the free energy [3], that they are characterized by an inequality expressing a balance between energy and entropy [4], etc. .... We want to add another characterization and this by means of the notion of detailed balance which is widely used in the physics literature [5, 6].

The detailed balance condition has a well-defined physical meaning, it expresses the duality of a jump process between two states with its inverse process.

This notion entered the mathematics literature under the name of a reversible process [7, 8]. We will continue to use the name detailed balance and give an independent definition. Our definition will also turn out to have a more universal character than the previous one.

Our main result is that we are able to construct explicitly an infinite set of detailed balance processes, and that we prove that any state which is invariant for all these processes is necessarily a DLR-state, even when the state or the potential is not translation invariant. This extends earlier results (see discussion). We also want to stress the extreme simplicity of the proof.

II. Detailed Balance Processes and DLR-Conditions

Consider the lattice Z^ν ($\nu = 1, 2, 3, \ldots$). To each site $j \in Z^\nu$ we associate a copy K_j of a compact set K. For any subset $X \subset Z^\nu$, denote $K_X = \prod_{j \in X} K_j$ and $C(X)$ the set of real