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The Yang-Mills Collective-Coordinate Potential

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Abstract. The potential of the pure Yang-Mills theory when quantized on the space of gauge fields modulo gauge transformations is computed. The large-N behaviour is given in terms of the Green's function for a scalar field in the adjoint representation.

1. Introduction

In a pure SU(N) Yang-Mills theory there are only two inherent parameters in which to expand: the coupling constant g and the group size N. In the $N \to \infty$ limit the perturbative quantum theory is known to simplify in the sense that expectation values of products of equal-time Wilson loop functionals factorize [1]. A mechanism for this reduction has recently been found and applied to simpler field theories with a large-N limit [2]. It is known as the collective-coordinate method and revolves around the fact that as N increases, the symmetry of theory increases. If only states which are invariant under the symmetry group (the group of local gauge transformations for Yang-Mills) are to be considered, then the collective coordinates are to parametrize group-inequivalent fields. When the Hamiltonian is written in terms of the collective coordinates an extra term appears, called the collective-coordinate potential (CCP). In the simpler models studied so far [3], as N increases and the coupling constants are suitably scaled the total potential is found to become sharply peaked around one point. Then the ground-state wave function will be peaked at this point and the expectation values of products of functions of the collective coordinates will factorize. In the case of large-N Yang-Mills such a point of peaking was termed the "master field" by Witten [1].

We attempt to give an intrinsic expression for the CCP in the sense that it depends directly on gauge-invariant properties for the gauge field and not on collective coordinates. More specifically, our expressions (47) and (49) for the total potential in the large-N limit involve only the covariant derivative of the Green's function for an adjoint scalar in a background field. The expression (39) for the CCP is exact for all N. These formulas may be more computationally tractable than previous expressions.