Commun. Math. Phys. 95, 247–255 (1984)

Communications in Mathematical Physics

Analytic Study of the Migdal-Kadanoff Recursion Formula

K. R. Ito*

ZiF, Bielefeld University, Wellenberg 1, D-4800 Bielefeld 1, Federal Republic of Germany

Abstract. After proposing lattice gauge field models in which the Migdal renormalization group recursion formulas are exact, we study the recursion formulas analytically. If D is less than 4, it is shown that the effective actions of D-dimensional $U(1)$ lattice gauge models are uniformly driven to the high temperature region no matter how low the initial temperature is. If the initial temperature is large enough, this holds for any D and gauge group G. These are also the cases for the recursion formulas of Kadanoff type. It turns out, however, that the string tension for $D=3$ obtained by these methods is rather big compared with the one already obtained by Mack, Göpfert and by the present author. The reason is clarified.

1. Introduction

A decade ago, Migdal [1] proposed renormalization group recursion formulas which are rather simple but are believed to approximate the real systems fairly well. After his works, Kadanoff [2] reconstructed similar recursion formulas by his decimation methods. The most surprising feature of these recursion formulas is that the recursion formulas of D-dimensional lattice gauge models are equivalent to those of $(D/2)$-dimensional statistical mechanical models (with nearest neighbor interactions). One advantage is that these recursion formulas have a closed form and then they may be solved analytically.

But it remains to see to what extent these approximate formulas are close to the real systems. We answer this question by solving the recursion formulas analytically. The answer is quite negative at least when D is less than 4: the effective dimensions are too close to two if $D<4$. Thus these methods give larger string tensions compared with the real ones.

Before writing down these formulas, we propose special lattice gauge field models in which the recursion formulas of these types are exact. This may help the

* On leave of absence from: Dept. of Maths. Bedford College, Regents Park, London NW 1, England

Address after 1.10.1984: Dept. of Maths. College of Liberal Arts, Kyoto University, Sakyo-ku, Kyoto 606, Japan