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Geometry of $N=1$ Supergravity

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Abstract. A new geometrical formalism is suggested for the non-minimal and alternative minimal supergravities. This formalism connects the constrained superspace formulations with the unconstrained ones and is based on the notion of induced geometry. The relevant mathematical technique is that of G-structures. A clear-cut geometrical content of the torsion and curvature constraints is revealed on the basis of a general theorem about the necessary and sufficient properties of induced geometry.

The $N=1$ supergravity can be formulated in superspace in a number of different ways. There is a whole family of supergravity theories, parametrized conveniently by a parameter ζ [1–3]. These superspace formulations correspond in components to different sets of auxiliary fields, minimal ($\zeta=\infty$), non-minimal ($\zeta \neq 1/3, 1, \infty$), and alternative minimal ($\zeta=1$) sets, that were discovered respectively in [4–6]. At the same time various geometrical approaches to superspace are known in all mentioned cases.

In this paper we continue the study of the geometry of $N=1$ supergravities using the framework of induced structures. (The general notion of the induced structure has been introduced in [7]. It describes the internal geometry of a surface inherited from the geometry of the ambient space). The case $\zeta=\infty$ was already investigated by means of these methods [7]. Recently it has been shown [8] that completely analogous constructions can be used in the description of $N=1$ super Yang-Mills theory coupled to supergravity. Our aim in the present paper is to look at the structure of supergravity from a general geometrical point of view. We shall show that the links between prepotentials and constrained supervielbeins in $N=1$ supergravity can be understood on the basis of a theorem about the necessary and sufficient properties of induced geometry. (The formulation and the proof of this theorem are given in [22], which contains also, as a corollary, a derivation of supergravity constraints.) The generality of the methods used allows us to expect further applications. The construction of action functionals is also considered. It is shown, in particular, that the "non-geometrical" action of the alternative