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Approach to Equilibrium for Locally Expanding Maps in $\mathbb{R}^k$

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Abstract. By using a well known technique from classical statistical mechanics of one-dimensional lattice spin systems we prove existence of an absolutely continuous invariant asymptotic measure for certain locally expanding maps T of the unit cube in $\mathbb{R}^k$. We generalize herewith in a certain sense the results of Lasota and Yorke on piecewise expanding maps of the unit interval to higher dimensions. We show a Kuzmin-type theorem for these systems from which exponential approach to equilibrium and strong mixing properties follow.

Introduction

There has been quite a lot of interest over the last years in a possible prediction of the long time behaviour of inherent stochastic deterministic systems. Many of these systems depend very sensitively on initial conditions and the smallest uncertainties grow in general exponentially fast. This makes any long term prediction of a single orbit practically impossible. From classical statistical mechanics where one handles systems with a huge number of degrees of freedom, one is familiar with another kind of description which is based on their statistical behaviour. It was quite a surprise that also systems with only a very few degrees of freedom should allow for such a description. But computer calculations, which up to now are the main tool for handling such nonlinear systems, showed that one has in general a well defined mean behaviour: time averages of most observables along single trajectories exist and do not depend on the specific trajectory. This suggests that many of these systems support very special measures for which some sort of generalized ergodic theorem is expected to hold. These measures are the so-called asymptotic measures.

From the physical point of view they just correspond to the well known Gibbs measures of classical statistical mechanics which also describe the long time behaviour of Hamiltonian systems. Contrary to the situation there for general dynamical systems, the asymptotic measures are hardly known and even their eixtence is a rather difficult problem, not to mention explicit analytic expressions.

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