Commun. Math. Phys. 94, 439–458 (1984)

Communications in Mathematical Physics
© Springer-Verlag 1984

The Ground State Energy of a Classical Gas*

Joseph G. Conlon**

Institut für Theoretische Physik, Universität Wien, Boltzmanngasse 5, A-1090 Wien, Austria

Abstract. In this paper we study the ground state energy of a classical gas. Our interest centers mainly on Coulomb systems. We obtain some new lower bounds for the energy of a Coulomb gas. As a corollary of our results we can show that a fermionic system with relativistic kinetic energy and Coulomb interaction is stable. More precisely, let $H_N(\alpha)$ be the N particle Hamiltonian

$$H_N(\alpha)=\alpha\sum_{i=1}^{N}(-\Delta_i)^{1/2}+\sum_{i<j}|x_i-x_j|^{-1}-\sum_{i,j}|x_i-R_j|^{-1}+\sum_{i<j}|R_i-R_j|^{-1},$$

where Δ_i is the Laplacian in the variable $x_i\in\mathbb{R}^3$ and $R_1,\ldots,R_N$ are fixed points in $\mathbb{R}^3$. We show that for sufficiently large α, independent of N, the Hamiltonian $H_N(\alpha)$ is nonnegative on the space of square integrable functions $\psi(x_1,\ldots,x_N)$, antisymmetric in the variables x_i, $1\leqq i\leqq N$.

Introduction

Consider the N particle Hamiltonian $H_N^R(\alpha)$ defined by

$$H_N^R(\alpha)=\alpha\sum_{i=1}^{N}(-\Delta_i)^{1/2}+\sum_{i<j}|x_i-x_j|^{-1}-\sum_{i,j}|x_i-R_j|^{-1}+\sum_{i<j}|R_i-R_j|^{-1},\quad(1.1)$$

where Δ_i is the Laplacian in the variable $x_i\in\mathbb{R}^3$ and $R_1,\ldots,R_N$ are fixed points in $\mathbb{R}^3$. We prove the following:

Theorem 1.1. *Let $H_N^R(\alpha)$ act on the space of square integrable functions $\psi(x_1,\ldots,x_N)$ on $\mathbb{R}^{3N}$, antisymmetric in the variables x_i, $1\leqq i\leqq N$. Then there exists a universal constant α such that $H_N^R(\alpha)\geqq 0$.*

---

* Research supported by grants from the Austrian National Science Foundation and University of Missouri research council

** Permanent address: Department of Mathematics, University of Missouri, Columbia, MO 65211, USA