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SU(N) Yang-Mills Solutions with Constant Field Strength on T^4

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Abstract. We study for T^4 the class of solutions to the SU(N) Yang-Mills equations with constant field strength. The fluctuation spectrum is explicitly calculated in terms of generalized Riemann theta functions. We show that if these solutions are stable, they are necessarily (anti)-selfdual, in which case we verify the index theorem.

1. Introduction

Euclidean solutions to the classical equations of motion (instantons), play an important role in the nonperturbative analysis of gauge theories. For S^4 the most general solution is known now (ADHM construction [1]) using advanced mathematical results. It was hoped that they could be used to understand confinement, however Coleman's argument shows that instantons have no effect on the Wilson loop, which is used to measure the static quark-antiquark potential [3].

For the torus this argument is no longer valid, because twisted boundary conditions [2] force electric and magnetic flux through the box. Nevertheless one will encounter as always severe infrared problems. Physical quantities are expressed in terms of the running coupling constant, which for small box size L is proportional to $(-\ln L)^{-1/2}$. So if L increases, the running coupling constant increases and perturbation theory breaks down, not only in the perturbative but also in the instanton sectors. Including instantons however can give an earlier signal for the crossover. Moreover the analysis of Lüscher [4] shows that the energy of the ground state is independent of the electric flux $\mathbf{e}$ (the central sectors) to all orders in perturbation theory. Confinement would be signalled by an energy difference, between the ground state levels in each central sector, proportional to L. Since "twisted" instantons lift the degeneracy they might be crucial to detect this behaviour. This work is intended as a first step in that direction.

We will concentrate on gauge fields with constant curvature. Such soltuions were already considered some time ago by 't Hooft [5] for SU(N) on T^4 with twisted boundary conditions. Only if the sides of the box representing T^4 satisfy certain relations, these solutions are (anti)-selfdual. For small groups and arbitrary compact manifolds [including SU(2) (SO(3)) bundles over T^4] stable extrema of the action are (anti)-selfdual [6]. It is therefore no surprise that in SU(2) an explicit calculation shows that constant solutions which are stable are (anti)-selfdual (the reverse is obvious).

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