

A Simplified $SO(6,2)$ Model of $SU(3)$

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Abstract. A new realization is obtained of the representation of $so(6,2)$ which has been shown recently by Flath and Biedenharn, and also by Bracken and MacGibbon, to define a model of $SU(3)$. In contrast to the realization in terms of six pairs of boson operators used previously, which involved cubic expressions, the new realization involves only quadratic expressions in eight pairs of boson operators, and is manifestly hermitian. Properties of this new “oscillator realization”, and in particular its advantages over the old realization, are discussed briefly. It is deduced that the representation of $so(6,2)$ is integrable to a unitary group representation.

Introduction

Following Bernšteĭn, Gel’fand and Gel’fand [1], a model of a compact group G is defined as a realization of a representation of G which contains in direct sum exactly one representative from each and every equivalence class of irreducible representations (irreps) of G .

Recently a remarkable model of $SU(3)$ has been discovered by Flath and Biedenharn [2–5] and also by Bracken and MacGibbon [6]. There exists a realization in terms of boson operators of an irrep of the Lie algebra $so(6, 2)$, which contains every distinct hermitian irrep of the Lie subalgebra $su(3)$ exactly once and so defines a model of $SU(3)$. Furthermore, basis elements of the $so(6,2)$ algebra are represented in this case by Wigner tensor operators (that is, tensor shift-operators) for $SU(3)$.

Flath and Biedenharn have emphasized that this model provides the framework for an elegant description of the algebra of $SU(3)$ tensor operators, including a complete resolution of the multiplicity problem for such operators. On the other hand, Bracken and MacGibbon have emphasized that this realization of $so(6,2)$ defines “creation and annihilation” operators which, when applied to a

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