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Study of the Iterations of a Mapping Associated to a Spin Glass Model

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Abstract. We study the iterations of the mapping

$$\mathcal{N}[F(s)]=\frac{(F(s))^2-(F(0))^2}{s}+(F(0))^2,$$

with the constraints $F(1)=1$, $F(s)=\sum a_n s^n$, $a_n \geqq 0$, and find that, except if $F(s)\equiv s$, $\mathcal{N}^k[F(s)]$ approaches either 0 or 1 for $|s|<1$ as $k\to\infty$.

I. *Introduction and Summary of the Results*

In a simplified version of a spin glass model [1] (CEGM), the probability distribution of the spin-spin interaction is given by a discrete set of coefficients a_n,

$$\sum_{n=0}^{\infty} a_n=1, \qquad a_n \geqq 0, \tag{1}$$

and after the operation of the renormalization group, this distribution is replaced by a new one. The operation is best described by writing the equation which gives the new generating function of the probabilities, $\mathcal{N}F$, in terms of the old one, F:

$$\mathcal{N}[F(s)]=\frac{(F(s))^2-(F(0))^2}{s}+(F(0))^2, \tag{2}$$

with $F(s)=\sum a_n s^n$. This mapping preserves conditions (1).

We want to study the iterations of (2) and find out what happens to

$$\mathcal{N}^k[F(s)]=\underbrace{\mathcal{N}[\mathcal{N}[\mathcal{N}\dots\mathcal{N}}_{k\text{ times}}[F(s)]]], \quad \text{for} \quad k\to\infty,$$

and see whether $\mathcal{N}^k F(s)$ approaches a limit or has a chaotic behavior. In the sequel, we use the abbreviation

$$\begin{aligned}\mathcal{N}^k[F(s)]&=F^{(k)}(s),\\ F^{(k)}(s)&=\sum a_n^{(k)} s^n.\end{aligned} \tag{3}$$