Commun. Math. Phys. 94, 289–291 (1984)

Communications in
Mathematical
Physics

Remark on the Continuity of the Density of States of Ergodic Finite Difference Operators

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Abstract. We give an elementary proof that for a large class of d-dimensional finite difference operators including tight-binding models for electron propagation and models for harmonic phonons with random masses or couplings, the integrated density of states is a continuous function of the energy.

Let us first consider the self-adjoint finite difference Schrödinger operator acting on $\ell^2(\mathbb{Z}^d)$ defined by

$$(H\psi)(x) = \sum_{\substack{y \in \mathbb{Z}^d \\ |y-x|=1}} \psi(y) + V(x)\psi(x), \qquad V(x) \in \mathbb{R}, \qquad x \in \mathbb{Z}^d \tag{1}$$

(if V is unbounded, the set of sequences ψ with finite support is a core for H). $P(]-\infty, E[)$ will be the associated spectral projection on the energy interval $]-\infty, E[$, and $P(\{E\})$ the projection on the eigenspace associated with E. We, furthermore, consider a probability measure μ on the potentials $\{V(x)\}_{x \in \mathbb{Z}^d}$, namely, a probability measure $\mathbb{R}^{\mathbb{Z}^d}$ with the σ-algebra generated by cylindrical events, and we suppose μ to be ergodic with respect to the translations of $\mathbb{Z}^d$. It is then known [1] that

$$k_\Lambda(E, H) = \frac{1}{|\Lambda|} \operatorname{Tr} P(]-\infty, E])\chi_\Lambda, \tag{2}$$

where χ_Λ stands for the characteristic function of a finite subset Λ of $\mathbb{Z}^d$, converges as $\Lambda \uparrow \mathbb{Z}^d$ for μ-a.a. potential $\{V(x)\}_{x \in \mathbb{Z}^d}$ to a non-random function

$$k(E) = \mathbb{E}_\mu(\langle \delta_0, P(]-\infty, E])\delta_0 \rangle), \tag{3}$$

where $\mathbb{E}_\mu$ denotes expectation with respect to the measure μ and δ_0 is the element located at 0 of the canonical basis of $\ell^2(\mathbb{Z}^d)$; $k(E)$ is the integrated density of states and can also be obtained by limits of systems enclosed in finite boxes.

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