

On Quantum Field Theory in Gravitational Background

Rudolf Haag, Heide Narnhofer*, and Ulrich Stein

II. Institut für Theoretische Physik, Universität Hamburg, D-2000 Hamburg 50,
Federal Republic of Germany

Abstract. We discuss quantum fields on Riemannian space-time. A principle of local definiteness is introduced which is needed beyond equations of motion and commutation relations to fix the theory uniquely. It also allows us to formulate local stability. In application to a region with a time-like Killing vector field and horizons it yields the value of the Hawking temperature. The concept of vacuum and particles in a non-stationary metric is treated in the example of the Robertson-Walker metric and some remarks on detectors in non-inertial motion are added.

I. Introduction

In the past decade there has been increasing interest in the interplay between the principles of general relativity and those of quantum physics. Some aspects arise already when one considers local quantum physics in a Riemannian space whose causal structure is described by a given, classical metric field $g_{\mu\nu}(x)$. Typical examples are the Hawking temperature of a black hole [1], the behaviour of accelerated detectors as discussed by Unruh [2] and the definition of particle states in an expanding universe. The essential phenomena and problems arising from the deviation of $g_{\mu\nu}$ from the Minkowski metric can be illustrated in these examples by quantum fields obeying linear field equations. We shall confine our discussion to this.

The first question is whether the quantum theory is unambiguously defined by the equation of motion and commutation relations without recourse to a special, distinguished state, the vacuum [2a]. In the Minkowski world this special state is tied to the time translation symmetry of the theory, a feature which is lost in general (there will usually be no global time-like Killing vector fields). To answer this question we have to make clear what we mean by “the theory.” Our customary

* Permanent address: Institut für Theoretische Physik, Universität Wien, Wien, Austria