

Classical and Quantum Mechanical Systems of Toda-Lattice Type

II. Solutions of the Classical Flows

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Abstract. Solutions to the classical periodic and non-periodic Toda lattice type Hamiltonian systems are expressed in terms of an Iwasawa-type factorization of a “large” Lie group. The scattering of these systems is determined in the non-periodic case. For the generalized periodic Toda lattices a generalization of Kostant’s formula is obtained using standard representations of affine Lie groups.

Contents

0. Introduction	178
1. Lie Group Factorizations and Lax Equations	
1.1. Factorizations and Flows.	180
1.2. Solution of Lax Equations	180
2. Solution of Lax Equations on \mathfrak{p}	
2.1. Lax Equations on Riemannian Symmetric Spaces	181
2.2. Asymptotic Behavior of Solutions and the QR Algorithm	182
2.3. Scattering	185
3. Integrable Hamiltonian Systems on Iwasawa Groups	
3.1. Solution to Euler Equations on \mathfrak{s}^*	186
3.2. J -Regular Orbits in \mathfrak{s}^*	187
3.3. Orbits of Toda Type.	191
3.4. Construction of Toda Orbits (Basic Examples).	191
3.5. Construction of Toda Orbits (Amalgamation)	194
3.6. Scattering on J -Regular Orbits	196
4. Hamiltonian Systems Associated with Affine Root Systems	
4.1. Lax Equations on Loop Groups.	198
4.2. Finite-Dimensional Subquotients of $\tilde{\mathfrak{g}}$	201
4.3. Geodesic Flow on B_k	203
4.4. Solution of Periodic Toda Lattices.	204

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