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Existence of Maximal Surfaces in Asymptotically Flat Spacetimes

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Abstract. We prove the existence of maximal surfaces in asymptotically flat spacetime satisfying an interior condition. This uses a priori estimates which can also be applied to prescribed mean curvature surfaces in cosmological spacetimes and the Dirichlet problem.

1. Introduction

Maximal surfaces are spacelike submanifolds of a Lorentzian manifold which locally maximize the induced area functional. This leads to a nonlinear elliptic equation which is interpreted geometrically as the vanishing of the mean extrinsic curvature. More generally, one considers surfaces of prescribed mean curvature. The main interest in such surfaces presently comes from general relativity, where there have already been many applications. For example, they have been used to prove positivity of mass [SY1], analyse the space of solutions of Einsteins equations [FMM] and in numerical integration schemes for Einsteins equations [P, ES]. Further references can be found in review papers such as [MT, ChY].

It is clear that a good understanding of the existence and regularity properties of such surfaces is needed. In this paper we prove the existence of asymptotically flat maximal surfaces in asymptotically flat spacetimes satisfying a uniformity condition in the interior (Theorem 5.4). Along the way we show that the Dirichlet problem in nonflat spacetimes is solvable (Theorem 4.2) and prove the existence of constant mean curvature surfaces in cosmological spacetimes (Theorem 4.1). The result for cosmological spacetimes was first proved by Claus Gerhardt, but our proof appears to be simpler.

These results hold under very general conditions. For example, the usual energy inequalities on the curvature tensor [HE] are not needed, the mean curvature can be nonconstant and singularities protected by barrier surfaces (i.e. crushing singularities [ES]) are permitted. The restrictions are that the spacetime admits a smooth time function and that some compactness condition is satisfied. This latter condition is needed to ensure that spacelike surfaces with given boundary do not reach arbitrarily far into the future (past). An instructive example