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Correlation Functions of the One-Dimensional Bose Gas in the Repulsive Case

V. E. Korepin

Leningrad Department of the V. A. Steklov Mathematical Institute, Fontanka, 27, SU-191011, Leningrad, USSR

Abstract. The one dimensional Bose gas is considered in the repulsive case. The ground state of the system is the Dirac sea with a finite density. The correlation function of the currents is presented in the form of the series, the n^{th} term being the contribution of n vacuum particles. In the strong coupling limit $c \to \infty$ the n^{th} term decreases as c^{-n}. In the weak coupling limit $c \to 0$ the series is also essentially simplified. The decomposition gives the uniform approximation in the distance between the currents. The arguments in favour of convergence of the series are given.

1. Introduction

In the present paper we consider the one dimensional Bose gas, which is equivalent to the quantum non-linear Schrödinger (NS) equation. We use the abbreviation NS to denote the model. Its Hamiltonian is equal to

$$\mathbf{H} = \int_0^L dx(\partial_x \psi^+ \partial_x \psi + c\psi^+\psi^+\psi\psi - h\psi^+\psi), \; [\psi(x), \psi^+(y)] = \delta(x-y).$$

We use the approach of paper [1], where the problem was imbedded in the quantum inverse scattering method (QISM) [2–4]. The notations and definitions of paper [1] are exploited also. The formulae of this paper are cited as (number of the formula) [1]. The NS model was studied in the thermodynamic limit in [5–7], where the ground state (the physical vacuum) was constructed. The system of transcendental equations (s.t.e.) (1.5) [1] for this state looks like:

$$\varphi_j = 2\pi j + \pi N, \; \varphi_j = \lambda_j L + \sum_{\substack{k=-N/2 \\ k \neq j}}^{N/2} \Phi(\lambda_j - \lambda_k), \; \Phi(\lambda) = i \ln\left(\frac{\lambda + ic}{\lambda - ic}\right). \tag{1.1}$$

The integer number j takes all the values in the interval $[-N/2, N/2]$. The number $(N+1)$(odd) is the number of the particles in the vacuum. In the thermodynamic limit $L \to \infty$, $N/L = \text{const}$ momenta λ_j—the solutions of the equations (1.1) fill the