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A New Approach to the Self-Dual Yang–Mills Equations

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Abstract. Inspired by Sato's new theory for soliton equations, we find a new approach to the self-dual Yang–Mills equations. We first establish a correspondence of solutions between the self-dual Yang–Mills equations and a new system of equations with infinitely many unknown functions. It then turns out that the latter equations can be easily solved by a simple explicit procedure. This leads to an explicit description of a very broad class of solutions to the self-dual Yang–Mills equations, and also to a construction of transformations acting on these solutions.

Introduction

Recently there has been significant progress due to Sato [1] in the theories of completely integrable systems. Further developments from slightly different aspects can be found in the papers of Date et al. [2], Mulase [3], Segal and Wilson [4], Ueno and Takasaki [5], etc. Sato's theory provides, in a unified framework, a new formulation of the "complete integrability" of various kinds of "soliton equations" (the KdV equation, the nonlinear Schrödinger equation, the Kadomtsev–Petviashvili equation, etc.), without using the conventional inverse scattering techniques including the Riemann–Hilbert problem. In his formulation all the information contained in a solition equation is "coded" into the time evolution of $\infty \times \infty$ matrices which are regarded as points of an infinite dimensinal Grassmann maniford, and in such a picture the structure of both solutions and transformations acting on them can be clearly described. In this paper we shall discuss a similar approach to the self-dual Yang–Mills equations.

In the four dimensional complex flat space $\mathbb{C}^4$ with coordinates $x = (y, z, \bar{y}, \bar{z})$, the (complexified) self-fual Yang–Mills equations with structure group $\mathrm{GL}(r, \mathbb{C})$

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