

An Alternate Version of Pirogov-Sinai Theory

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Abstract. A new approach to the Pirogov-Sinai theory of phase transitions is developed, not employing the contour models with a parameter. The completeness of the phase diagram is proven.

Introduction

The aim of the following note is twofold.

i) The theory of Pirogov and Sinai describing the behaviour of phase diagrams of lattice models with discrete spins is based on the notion of a contour model and a contour model with a parameter. The notion of a contour model goes back to Minlos and Sinai's detailed study of the Ising model [1].

If one is interested in the investigation of the behaviour of the point of the maximal number of phases only, there is no need for introducing the models with parameter in the Pirogov-Sinai (PS) theory. This was, historically, the first case studied, but it had many of the essential features of the theory.

It is the aim of the first part of this note to show that even the full phase diagram can be constructed without the use of models with a parameter. Instead, we will introduce an auxiliary "metastable" contour model, not appearing in the usual PS theory. It seems that this approach is (intuitively, at least) simpler than the original approach [2]. It gives some additional information, too.

ii) There are some results of Martirosian (see e.g. [4]) concerning the completeness of the phase diagram constructed by the PS method. The full answer is given here. The proof is simple and uses the same tools which were developed in i). In particular, we emphasise the technical main lemma which is crucial in the proofs of both parts of this paper.

The organization of the paper is as follows:

Section 1 is devoted to the questions mentioned in i). We suppose some familiarity with PS ideas and notions like contours, contour models etc. For these notions which are only sketched the standard reference is [2] or [3], but no technical details of [2] are used in this paper.