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Markov Dilations and Quantum Detailed Balance

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Abstract. We construct quantum stochastic processes whose multi-time correlation functions, with suitable time ordering, can be obtained from a quantum dynamical semigroup. We prove that such a process defines a stationary Markov dilation of the associated semigroup if and only if (up to technicalities) the semigroup satisfies the quantum detailed balance condition with respect to its stationary state.

1. Introduction

Quantum dynamical semigroups have been widely used in the last ten years to describe irreversible time evolutions of open systems. The connection with the underlying Hamiltonian dynamics of isolated systems has been investigated in both directions. For a class of models of open systems, it has been proved that the reduced evolution converges to a dynamical semigroup in the *weak* or the *singular coupling limit* [1, 2]. Conversely, it has been shown that any dynamical semigroup Φ_t on a C^*-algebra $\mathscr{B}$ admits a *unitary dilation*, consisting of an embedding j_0 of $\mathscr{B}$ into another C^*-algebra $\mathscr{A}$, a group α_t of *-automorphisms of $\mathscr{A}$ and a norm one projection E_0 of $\mathscr{A}$ onto j_0 ($\mathscr{B}$) such that $\Phi_t = j_0^{-1} E_0 \alpha_t j_0$ for all t in $\mathbb{R}^+$ [3–5]; however, the unitary dilation is far from unique.

More recently, it was recognized that a tighter connection between irreversible evolution and underlying Hamiltonian dynamics could be obtained by the consideration of multi-time correlation functions [6]. This leads to the idea of a *quantum stochastic process* [7–9], which should be determined up to equivalence by its multi-time correlations, much in the same way as a stochastic process in the sense of Doob is determined by its finite-dimensional joint distributions. Then it becomes possible to require that the unitary dilation defines a Markov process. The convergence of multi-time correlations in the weak or the singular coupling limit was proved by Dümcke [10, 11] for a class of models; the converse problem of Markov dilations has been investigated by Kümmerer and Schröder [12–14, 11]. Related works include the generalized K-flows of Emch et al. [15, 16] and the