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Ergodic Properties of the Lozi Mappings

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Abstract. In this paper, we construct the Bowen-Ruelle measure for the Lozi mapping, an almost everywhere hyperbolic diffeomorphism of the plane. We also derive some of its properties which are similar to those of an axiom A system.

I. Introduction

The Lozi mapping T is a homeomorphism of $\mathbb{R}^2$ given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \to \begin{pmatrix} 1+y-a|x| \\ bx \end{pmatrix}.$$

For some values of a and b, Lozi [Lo] observed complicated behaviour for the trajectories of this system. For $b=0.5$ and $a=1.7$ one observes numerically a strange attractor, which is very similar to the attractor of the Henon map [He]. The main advantage of the Lozi map over the Henon map is that one can prove hyperbolicity without much effort. This is the main reason why so little is known for the Henon map, where hyperbolicity is believed to occur only on Cantor-like sets of parameters. Our opinion is that the Lozi mapping is an intermediate stage between the Axiom A dynamical systems and more complicated systems like the Henon map. As we shall see below, its dynamical structure is more complicated than in the Axiom A systems, although some detailed ergodic properties are the same. The Lozi map is rather similar to Sinai's billiards, and in this article, we shall use this analogy. In particular, the discontinuity of the differential allows the uniform hyperbolicity as in the billiards case. A proof of hyperbolicity for the Lozi map was first given by Misiurewicz [M]. He also derived many important consequences which will be described below.

This article is devoted to the investigation of the metric properties of the Lozi map. In the next paragraph, we briefly describe some properties of the map which will be needed later on. Most of them were known before. In the third paragraph we construct an invariant measure; its ergodic properties (absolute continuity with