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A Spin-Glass Model with Random Couplings

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Abstract. We define a frustrated spin-glass model for which the Migdal-Kadanoff renormalization group is exact. Our model has random couplings, and the renormalization group acts on these. We study the high and low temperature phases of the model, exhibit a critical fixed point (in high dimension), and show that the Edwards-Anderson parameter takes a non-zero value in the low-temperature phase.

Introduction

In this paper, we analyze a model of spin-glasses which is similar to one studied before in a joint paper with Glaser and Martin [CEGM]. While that paper was concerned with what might be called the low-dimensional case, the present paper deals with the high-dimensional limit. In order to make this paper readable independently of [CEGM], we have chosen to repeat more or less verbally some of the introductory parts of that paper. We want to view the spin-glass problem as a problem of *random variables* describing random couplings. In particular, we are interested in the behavior of the effective random coupling under a *change of scale*. This will lead us naturally to a renormalization group (RG) approach.

This description will become exact in the hierarchical approximation described below, see also [GK], and we shall describe and study some aspects of the corresponding models which are random versions of a Migdal-Kadanoff type recursion relation [K, M]. Alternately, our approach leads to a study of non-independent (but *not* strongly coupled (mean-field) [SK]) random variables, and our results can be viewed as an example of non-trivial behavior in this field of mathematics.

The purpose of our paper is to describe and analyze a class of such models, and, in particular, to study the "evolution" of the effective random coupling as a function of the size of the lattice (Sects. 2–5).

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