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Determination of the Scattering Amplitude

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Abstract. The problem to determine the elastic scattering amplitude from the differential cross-section by the unitarity equation is reexamined. We prove that the solution is unique and can be determined by a convergent iteration if the parameter $\lambda = \sin\mu$ of Newton and Martin is bounded by $\lambda < \lambda_2 \simeq 0.86$. The method is based on a fixed point theorem for holomorphic mappings in a complex Banach space.

1. Introduction

The problem to determine the scattering amplitude when the differential cross-section is given by the integral equation of unitarity has been investigated for over 20 years [1–6] (a comprehensive list of references can be found in [7]). The unitarity equation for the elastic scattering amplitude of equal (pseudo-)scalar particles

$$F(x) = |F(x)|e^{i\varphi(x)} = G(x)e^{i\varphi(x)} \tag{1}$$

imposes a nonlinear constraint on the phase function $\varphi(x)$,

$$G(\mathbf{n}_1 \cdot \mathbf{n}_2) \cdot \sin\varphi(\mathbf{n}_1 \cdot \mathbf{n}_2) = \frac{1}{4\pi} \int d\Omega_{\mathbf{n}} G(\mathbf{n}_1 \cdot \mathbf{n}) G(\mathbf{n}_2 \cdot \mathbf{n}) \cos[\varphi(\mathbf{n}_1 \cdot \mathbf{n}) - \varphi(\mathbf{n}_2 \cdot \mathbf{n})], \tag{2}$$

where $x = \mathbf{n}_1 \cdot \mathbf{n}_2 = \cos\Theta \in [-1, +1]$ is the cosine of the scattering angle and $\mathbf{n}$ the unit vector in the direction of the momentum. The modulus $G(x) = |F(x)|$ is given by the differential cross-section. The characteristic quantity of this equation is the functional

$$\lambda = \sup_{\mathbf{n}_1, \mathbf{n}_2} \frac{1}{4\pi |F(\mathbf{n}_1 \cdot \mathbf{n}_2)|} \int d\Omega_{\mathbf{n}} |F(\mathbf{n}_1 \cdot \mathbf{n})| \, |F(\mathbf{n}_2 \cdot \mathbf{n})|. \tag{3}$$

In the following we assume that $|F(x)|$ is a continuous function without zeros. Hence the supremum (3) is a finite number. The mere existence of a continuous

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