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Integrability Conditions for Killing Spinors

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Abstract. The conditions for the existence of solutions of $D_\mu\eta = \pm c\gamma_\mu\eta$ are discussed. In general, it is not sufficient to consider only the first integrability condition $[D_\mu, D_\nu]\eta = -2c^2\gamma_{\mu\nu}\eta$; in particular, the second integrability condition is needed to explain why, in certain cases, only for one choice of sign does a solution exist. The Killing spinor-tensors, as defined by Walker and Penrose, are shown to be the spinorial equivalent of conformal Killing tensors. Their relationship to the Killing spinors and spinor-vectors used in supergravity, is given.

In supergravity, Killing spinors are very useful in analyzing the particle spectrum of Kaluza-Klein theories. They also determine the number N of supersymmetries. Killing spinors may be defined [1] to be solutions of

$$D_\mu\eta = \pm c\Gamma_\mu\eta\,, \tag{1}$$

where c is a constant related to the curvature of the background. The naive integrability condition for this equation [1, 2] reads

$$[D_\mu, D_\nu]\eta = \tfrac{1}{4}R_{\mu\nu mn}\Gamma^{mn}\eta = -2c^2\Gamma_{\mu\nu}\eta\,, \tag{2}$$

and is clearly insensitive to the sign in (1). It is known that on the round S_7 there are 8 solutions for either choice of sign, because these solutions have been explicitly constructed [3]. However, on the squashed S_7, an explicit construction [4] showed that there exists only one solution and for only one choice of sign. Depending on the sign in the Freund-Rubin ansatz [5, 6],

$$F_{\mu\nu\varrho\sigma} = \pm e\varepsilon_{\mu\nu\varrho\sigma}\,, \tag{3}$$

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