Commun. Math. Phys. 93, 259–275 (1984)

Communications in
Mathematical
Physics
© Springer-Verlag 1984

# Skeleton Inequalities and the Asymptotic Nature of Perturbation Theory for $\Phi^4$-Theories in Two and Three Dimensions

Anton Bovier and Giovanni Felder

Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

**Abstract.** We use the polymer representation of $\phi^4$-quantum field theories to prove an infinite family of correlation inequalities, called "skeleton inequalities", for the $2n$-point Green's functions. As an application, we show that they imply that Feynman perturbation theory is asymptotic in less than four dimensions.

## I. Introduction

Recently there has been a revival of interest in Symanzik's polymer representation [1] of quantum field theories. The probably most important results of this development so far have been the proofs of the triviality of the continuum limits of the Ising- and $\phi^4$-models in dimensions larger then four, due to Aizenman [2] and Fröhlich [3, 4].

In recent papers, Brydges, Fröhlich and Sokal [5, 6] have shown, however, that the polymer representation may also be useful to study the theory in lower dimensions. Their main result was a new, very simple proof of the existence and nontriviality of the continuum limit of $\phi^4$ in two and three dimensions. The new proof of Brydges et al. rests on new correlation inequalities, called "skeleton inequalities", which may be described as follows.

Call a full skeleton amplitude a Feynman diagram without selfenergy insertions where all the lines stand for full propagators. The "skeleton series" is the power series in $(-\lambda)$ with coefficients given by the full skeleton amplitudes associated with the skeletons of perturbation theory. Then, the partial skeleton series to even (odd) order are rigorous upper (lower) bounds for the corresponding Green's functions.

In [5] this conjecture has been proven up to order $n=2$ in $(-\lambda)$. In the present paper we give a complete proof to all orders in $(-\lambda)$. As an application we will use these bounds to obtain a new proof that perturbation theory gives asymptotic expansions for the continuum Green's functions for the one and two-component $\phi^4$ theory in dimensions less than four. Again, this is a known result [8, 9], but our