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On Absence of Diffusion near the Bottom of the Spectrum for a Random Schrödinger Operator on $L^2(\mathbb{R}^\nu)^+$

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Abstract. We consider a random Schrödinger operator on $L^2(\mathbb{R}^\nu)$ of the form $H_\omega = -\Delta + V_\omega$, $V_\omega(x) = \Sigma\chi_{C_i}(x)q_i(\omega)$, $\{C_i\}$ being a covering of $\mathbb{R}^\nu$ with unit cubes around the sites of $\mathbb{Z}^\nu$ and $\{q_i\}$ i.i.d. random variables with values in $[0,1]$. We assume that the q_i's are continuously distributed with bounded density $f(q)$ and that $0 < P(q_0 < \frac{1}{2}) = \alpha < 1$. Then we show that an ergodic mean of the quantity $\langle \int dx|x|^2|(\exp(itH_\omega)\Phi)(x)|^2\rangle t^{-1}$ vanishes provided $\Phi = g_E(H_\omega)\Psi$, where Ψ is well-localized around the origin and g_E is a positive C^∞-function with support in $(0, E)$, $E \leqq E^*(\alpha, |f|_\infty)$. Our estimate of $E^*(\alpha, |f|_\infty)$ is such that the set $\{x\in\mathbb{R}^\nu | V_\omega(x) \leqq E^*(\alpha, |f|_\infty)\}$ may contain with probability one an infinite cluster of cubes $\{C_i\}$ which are nearest neighbours. The proof is based on the technique introduced by Fröhlich and Spencer for the analysis of the Anderson model.

Section 1. Introduction

Let us consider a quantum mechanical particle moving in $\mathbb{R}^\nu$ and interacting with a random potential V_ω given by

$$V_\omega(x) = \sum_{i\in\mathbb{Z}^\nu} \chi_{C_i}(x)q_i(\omega). \tag{1.1}$$

Here $C_i = \{x\in\mathbb{R}^\nu | -\frac{1}{2} < x_j \leqq \frac{1}{2}; j = 1, \ldots, \nu\} + i$ and $\{q_i(\omega)\}_{i\in\mathbb{Z}^\nu}$ are independent identically distributed (i.i.d.) random variables with values in $[0,1]$ such that $P(q_0(\omega)\in[a,b]) = \int_a^b dq f(q)$, $|f|_\infty \equiv \text{ess sup}|f| < \infty$ and $0 < P(q_0 \leqq \frac{1}{2}) = \alpha < 1$. We are interested in the spectral properties of the corresponding random Hamiltonian H_ω on $L^2(\mathbb{R}^\nu)$

$$H_\omega = -\Delta + V_\omega, \tag{1.2}$$

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