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The Soliton Correlation Matrix and the Reduction Problem for Integrable Systems*

J. Harnad[1], Y. Saint-Aubin[2], and S. Shnider[1]

1 Centre de recherche de mathématiques appliquées, Université de Montréal, Montréal H3C 3J7, Canada, and
Department of Mathematics, McGill University, Montréal H3A 2T8, Canada
2 Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Abstract. Integrable $1+1$ dimensional systems associated to linear first-order matrix equations meromorphic in a complex parameter, as formulated by Zakharov, Mikhailov, and Shabat [1–3] (ZMS) are analyzed by a new method based upon the "soliton correlation matrix" (M-matrix). The multi-Bäcklund transformation, which is equivalent to the introduction of an arbitrary number of poles in the ZMS dressing matrix, is expressed by a pair of matrix Riccati equations for the M-matrix. Through a geometrical interpretation based upon group actions on Grassman manifolds, the solution of this system is explicitly determined in terms of the solutions to the ZMS linear system. Reductions of the system corresponding to invariance under finite groups of automorphisms are also solved by reducing the M-matrix suitably so as to preserve the class of invariant solutions.

1. Introduction

Consider the pair of differential equations

$$\left.\begin{aligned} \psi_\xi &= U\psi\,, \\ \psi_\eta &= V\psi\,, \end{aligned}\right\} \qquad (1.1)$$

where $U(\lambda,\xi,\eta)$, $V(\lambda,\xi,\eta)$, and $\psi(\lambda,\xi,\eta)$ are $n\times n$ matrix functions depending on a complex parameter λ, with ψ assumed invertible, and U and V meromorphic in λ with fixed poles on the Riemann sphere at $\{\alpha_r\}_{r=1,\ldots,l}$, $\{\beta_r\}_{r=1,\ldots,m}$ respectively. Such systems were introduced and studied by Zakharov, Mikhailov, and Shabat through the use of the "dressing method" [1–3]. Expressing the λ dependence of U and V explicitly as:

$$\left.\begin{aligned} U &= A_0 + \sum_{rs} \frac{A_r^s}{(\lambda-\alpha_r)^s}\,, \\ V &= B_0 + \sum_{rs} \frac{B_r^s}{(\lambda-\beta_r)^s}\,, \end{aligned}\right\} \qquad (1.2)$$

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