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Uniform Boundedness of Conditional Gauge and Schrödinger Equations

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Abstract. We prove that for a bounded domain $D \subset R^n$ with C^2 boundary and $q \in K_n^{\mathrm{loc}}$ $(n \geqq 3)$ if $E^x \exp \int_0^{\tau_D} q(x_t)dt \not\equiv \infty$ in D, then

$$\sup_{\substack{x \in D \\ z \in \partial D}} E_z^x \exp \int_0^{\tau_D} q(x_t)dt < +\infty \quad (\{x_t\}: \text{Brownian motion}).$$

The important corollary of this result is that if the Schrödinger equation $\frac{\Delta}{2}u + qu = 0$ has a strictly positive solution on D, then for any $D_0 \subset\subset D$, there exists a constant $C = C(n, q, D, D_0)$ such that for any $f \in L^1(\partial D, \sigma)$, ($\sigma$: area measure on ∂D) we have

$$\sup_{x \in D_0} |u_f(x)| \leqq C \int_{\partial D} |f(y)| \sigma(dy),$$

where u_f is the solution of the Schrödinger equation corresponding to the boundary value f.

To prove the main result we set up the following estimate inequalities on the Poisson kernel $K(x, z)$ corresponding to the Laplace operator:

$$C_1 \frac{d(x, \partial D)}{|x-z|^n} \leqq K(x, z) \leqq C_2 \frac{d(x, \partial D)}{|x-z|^n}, \quad x \in D, \quad z \in \partial D,$$

where C_1 and C_2 are constants depending on n and D.

Let D be a bounded domain in R^n $(n \geqq 3)$ with C^2 boundary, $(x_t, t > 0)$ be the Brownian motion and $\tau_D = \inf(t > 0 : x_t \notin D)$. According to Doob [3], for any positive harmonic function h on D, h-conditioned Brownian motion in D is

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