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Removable Singularities of Coupled Yang-Mills Fields in R^3

L. M. Sibner[1,2,*] and R. J. Sibner[1,3,**]

1 Mathematics Institute, University of Bonn, D-5300 Bonn, Federal Republic of Germany
2 Polytechnic Institute of New York, Brooklyn, NY 11201, USA
3 City University of New York, Brooklyn College, Brooklyn, NY 11210, USA

Abstract. We consider isolated point singularities of the coupled Yang-Mills equations in R^3. Under appropriate conditions on the curvature and the Higgs field, a removable singularity theorem is proved.

Introduction

The original removable singularity theorem of Uhlenbeck [19] in R^4, states that apparent point singularities in *finite action pure* Yang-Mills fields may be removed by a gauge transformation. Uhlenbeck's theorem was extended by Parker [13] to *coupled* Yang-Mills fields in R^4.

In R^3, finite action is too stringent a condition and may be relaxed to the assumption that the solution (i.e., the curvature) is in $L^{3/2}$. In recent work [17], it was shown that point singularities of solutions in $L^{3/2}$ of the *pure* Yang-Mills equations are removable.

In the following, we consider the *coupled* Yang-Mills equations in R^3. From the point of view of mathematical physics, our equations describe the *Higgs model* and have been studied extensively by Jaffe and Taubes [11]. We prove an isolated removable singularity theorem for solutions of these equations. The *sign* of the dominant lower order non-linear term plays a crucial role in this problem. In one case, no assumptions whatsoever are needed on the Higgs field to remove the singularity. In the other, a little more smoothness than is expected is required and an example of a singular solution is given which shows that the requirement is necessary. In both cases, we assume that the curvature is in $L^{3/2}$.

To prove the theorem, we first show that the Higgs field is bounded. This implies that its covariant derivative is square integeable and satisfies a strong growth condition on small balls about the puncture. This is then used to show that

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