Commun. Math. Phys. 92, 525–530 (1984)

Communications in Mathematical Physics

A Uniform Bound on Trace $(e^{t\Delta})$ for Convex Regions in R^n with Smooth Boundaries

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Abstract. We prove a bound (uniform in $t>0$) on trace $(e^{t\Delta})$ for convex domains in R^n with bounded curvature.

1. Introduction

Let D be a bounded domain in R^n with a smooth boundary ∂D. Let $\lambda_1 > \lambda_2 \geqq \lambda_3 \geqq \ldots$ be the eigenvalues of the eigenvalue problem

$$\Delta\phi = \lambda\phi \quad \text{on } D, \tag{1}$$

and

$$\phi = 0 \quad \text{on } \partial D. \tag{2}$$

It is well known that

$$Z(t) = \text{trace}(e^{t\Delta}) = \sum_{j=1}^{\infty} e^{t\lambda_j} \tag{3}$$

exists for all $t>0$, and that $Z(t)$ has an asymptotic expansion [1] of the form

$$Z(t) - \frac{1}{(4\pi t)^{n/2}} \cdot \sum_{k=0}^{K} c_k t^{k/2} = O(t^{(K-n+1)/2}), \qquad t \to 0. \tag{4}$$

The coefficients c_0, c_1, and c_2 have been calculated by McKean and Singer [4]. They depend on the geometrical properties of the domain D. For example

$$c_0 = |D| = \text{volume of } D, \tag{5}$$

and

$$c_1 = -\frac{\sqrt{\pi}}{2}|\partial D| = -\frac{\sqrt{\pi}}{2} \cdot \text{surface area of } \partial D. \tag{6}$$

In the special case of a two-dimensional domain ($n=2$) the coefficients $c_0, c_1, \ldots, c_6$ are known [2–7].