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Classical and Quantum Algebras of Non-Local Charges in σ Models

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Abstract. We investigate the algebras of the non-local charges and their generating functionals (the monodromy matrices) in classical and quantum non-linear σ models. In the case of the classical chiral σ models it turns out that there exists no definition of the Poisson bracket of two monodromy matrices satisfying antisymmetry and the Jacobi identity. Thus, the classical non-local charges do not generate a Lie algebra. In the case of the quantum $O(N)$ non-linear σ model, we explicitly determine the conserved quantum monodromy operator from a factorization principle together with $\mathscr{P}$, $\mathscr{T}$, and $O(N)$ invariance. We give closed expressions for its matrix elements between asymptotic states in terms of the known two-particle S-matrix. The quantum R-matrix of the model is found. The quantum non-local charges obey a quadratic Lie algebra governed by a Yang-Baxter equation.

I. Introduction

The notion of complete integrability in field theory involves the existence of an infinite number of commuting conserved quantities. In addition to these usually local quantities, some models possess an infinite number of non-local conserved charges which do not commute among themselves. This raises the important question whether the integrability of such field theories can be related to the existence of an infinite dimensional non-abelian dynamical symmetry algebra. For finitely many degrees of freedom dynamical symmetries are well known (e.g. the Coulomb problem and the harmonic oscillator). In field theory, the non-linear σ models are good candidates to possess this kind of structure.

To construct such a dynamical algebra one must find the Poisson brackets of non-local charges [1–4] in the classical field theory and the corresponding commutators in the quantum field theory. The monodromy matrix of the associated linear system [3–5] (Lax pair) serves as the generating functional of the non-local charges. This is a system of linear differential equations having the field

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