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Localization of Random Walks in One-Dimensional Random Environments

A. O. Golosov

Chair of Probability Theory, Department of Mechanics and Mathematics, Moscow State University, SU-117234 Moscow, USSR

Abstract. We consider a random walk on the one-dimensional semi-lattice $\mathbb{Z}_+ = \{0, 1, 2, \ldots\}$. We prove that the moving particle walks mainly in a finite neighbourhood of a point depending only on time and a realization of the random environment. The size of this neighbourhood is estimated. The limit parameters of the walks are also determined.

1. Introduction. Formulation of the Problem and Results

Let us consider a sequence $\mathscr{A} = \{(q(x), r(x), p(x));\ x \in \mathbb{Z}_+ = \{0, 1, 2, \ldots\}\}$ of random three-dimensional vectors whose components are non-negative, and $q(0) = 0$, $q(x) + r(x) + p(x) = 1$ for any $x \in \mathbb{Z}_+$. We shall call such a sequence a *random environment*. A random process $(x(t) : t \in \mathbb{Z}_+)$ will be called a *random walk in the random environment* $\mathscr{A}$ if the conditional distribution of $(x(t) : t \in \mathbb{Z}_+)$ under the condition that $\mathscr{A}$ is fixed is the distribution of the Markov chain whose phase space is $\mathbb{Z}_+$, initial state is 0, and probabilities of transitions from x to $x-1$, x, $x+1$ are $q(x)$, $r(x)$, $p(x)$, respectively; $x \in \mathbb{Z}_+$. We shall denote by $P(\cdot | \mathscr{A})$ probabilities of events depending on random walks if a realization of the random environment $\mathscr{A}$ is fixed. Probabilities of events calculated without the assumption that the random environment $\mathscr{A}$ is fixed (including events connected with any properties of the random environment) will be denoted by $P(\cdot)$.

We assume that the random vectors $(q(x), r(x), p(x))$ are mutually independent for different $x \in \mathbb{Z}_+$, $(q(x), p(x))$ are identically distributed for $x \geqq 1$, and $r(x)$ are identically distributed for $x \in \mathbb{Z}_+$. Moreover, we assume that the sequences of random variables $(r(x) : x \in \mathbb{Z}_+)$ and $(q(x)/p(x) : x \geqq 1)$ are independent, $E \ln(q(x)/p(x)) = 0$, $E(\ln(q(x)/p(x)))^2 = \sigma^2 \in (0, +\infty)$, $E(1 - r(x))^{-1} < +\infty$, $P\{r(x) > 0\} > 0$. Sinai [1] proved that for random walks in similar random environments with the phase space $\mathbb{Z} = \{\ldots, -1, 0, 1, 2, \ldots\}$ one can construct random variables $m(t)$ $(t \in \mathbb{Z}_+)$ depending only on t and a random environment such that $x(t) - m(t) = o(\ln^2 t)$ (in probability) as $t \to +\infty$, and there exists the limit distribution of $m(t) \cdot (\ln t)^{-2}$ as $t \to +\infty$ which coincides with that of $x(t) \cdot (\ln t)^{-2}$. An