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Non-Abelian Bosonization in Two Dimensions

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Abstract. A non-abelian generalization of the usual formulas for bosonization of fermions in $1+1$ dimensions is presented. Any fermi theory in $1+1$ dimensions is equivalent to a local bose theory which manifestly possesses all the symmetries of the fermi theory.

One of the most startling aspects of mathematical physics in $1+1$ dimensions is the existence of a (non-local) transformation from local fermi fields to local bose fields. Thus, consider the theory of a massless Dirac fermion:

$$\mathscr{L}_D = \bar{\psi} i \partial\!\!\!/ \psi . \tag{1}$$

This theory is equivalent [1] to the theory of a free massless scalar field:

$$\mathscr{L}_S = \tfrac{1}{2} \partial_\mu \phi \partial^\mu \phi . \tag{2}$$

The fermi field ψ has a relatively complicated and non-local expression [2] in terms of ϕ. However, fermion bilinears such as $\bar{\psi}\gamma_\mu\psi$ or $\bar{\psi}\psi$ take a simple form in the bose language. For example, the current $J_\mu = \bar{\psi}\gamma_\mu\psi$ becomes in terms of ϕ

$$J_\mu = \frac{1}{\sqrt{\pi}} \varepsilon_{\mu\nu} \partial^\nu \phi . \tag{3}$$

Similarly the chiral densities $\mathcal{O}_\pm = \bar{\psi}(1 \pm \gamma_5)\psi$ become

$$\mathcal{O}_\pm = M \exp \pm i \sqrt{4\pi} \phi , \tag{4}$$

where the value of the mass M depends on the precise normal ordering prescription that is used to define the exponential in (4).

By means of formulas like (3) and (4), the equivalence between the free Dirac theory and the free scalar theory can be extended to interacting theories. A perturbation of the free Dirac Lagrangian can be translated, via (3) and (4), into an equivalent perturbation of the free scalar theory. This procedure is remarkably

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