Commun. Math. Phys. 92, 269–294 (1983)

Communications in Mathematical Physics

Debye-Hückel Theory for Charge Symmetric Coulomb Systems

Tom Kennedy*

Department of Mathematics, University of Virginia, Charlottesville, VA 22903, USA

Abstract. It is proven that the pressure, density and correlation functions of a classical charge symmetric Coulomb system are asymptotic as the plasma parameter ε tends to zero to the approximations predicted by the Debye-Hückel theory. These approximations consist of the ideal gas term plus a term of one lower order in ε. The sine-Gordon transformation and some new correlation inequalities for the associated functional integrals are used.

1. Introduction

We study a classical charge symmetric system in three dimensions in the limit that ε tends to zero. ε is the plasma parameter

$$\varepsilon = \beta / \ell_D, \tag{1.1}$$

where β is the inverse temperature, and ℓ_D is the Debye length

$$\ell_D = (2\beta z)^{-1/2}. \tag{1.2}$$

z is the chemical activity. Debye and Hückel [4] gave a non-rigorous study of this limit. We will prove that certain predictions of their theory are rigorously correct in this limit.

The Debye-Hückel theory gives an approximation for the pressure P as a function of the density σ

$$\frac{1}{kT} P \simeq 2\sigma - \frac{1}{24\pi} \ell_D^{-3}. \tag{1.3}$$

(For example, see p. 229 of [11].) We work in the grand canonical ensemble, so the pressure and density are both functions of z and β. We will show that as ε tends to zero the pressure and density are asymptotically given by

$$\frac{1}{kT} P \sim 2z + \frac{1}{12\pi} \ell_D^{-3}, \quad \sigma \sim z + \frac{1}{16\pi} \ell_D^{-3}. \tag{1.4}$$

* Research in partial fulfillment of the requirements of the Ph.D. degree at the University of Virginia