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Convergent Expansions for Tunneling

J. M. Combes*, P. Duclos*, and R. Seiler**

Centre de Physique Théorique, CNRS – Luminy, Case 907, F-13288 Marseille Cedex 9, France

Abstract. A new method to compute effects of tunneling in one-dimensional multiple well is developed. A tunneling parameter built with physical quantities is introduced to measure the splitting between eigenvalues due to tunneling. These splittings are given by convergent series in term of this tunneling parameter for a wide class of double well.

I. Introduction

This is our second article in a series about the classical limit of one dimensional Schrödinger operators with multiple well potentials. The main subject of this article is tunneling. We shall use some of the results proved in the first paper of this series, referred to as [CDS 1]; it concerns the harmonic approximation, the exponential decay of eigenfunctions and asymptotic series for eigenvalues in the classical limit.

The third paper will treat tunneling in the case of shape resonances.

To describe more precisely what we mean by tunneling let us consider the Schrödinger operator

$$H(k) = -k^4 \Delta + V \tag{1}$$

on the interval Ω of the real line $\mathbb{R}$, where V is a potential function with an at most doubly degenerate absolute minimum v_0. Typical graphs of V for $v_0 = 0$ are shown in Fig. 1.

In the following we state the precise assumptions on V:

Assumptions on V. Let Ω be an arbitrary open interval and let V satisfy

i) $V \in L^1_{\text{loc}}(\Omega)$.

ii) V has an absolute minimum v_0 which can be at most degenerate of order 2.

* PHYMAT, Département de Mathématiques, Université de Toulon et du Var, and Centre de Physique Théorique, CNRS, F-13288 Marseille, France

** Institut für Theoretische Physik, Freie Universität, D-1000 Berlin