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Particle Spin From Representations of the Diffeomorphism Group

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Abstract. The semidirect product $\mathscr{S} \wedge \mathscr{K}$ of Schwartz' space $\mathscr{S}$ of functions on $\mathbb{R}^3$ with the group $\mathscr{K}$ of diffeomorphisms of $\mathbb{R}^3$ provides a model for quantum theory based on local currents. Certain unitary representations of $\mathscr{K}$ are induced by representations of $\overline{SL(3,\mathbb{R})}$. From the local currents in these representations, we construct the generators of local rigid rotations, with respect to which the Hilbert space decomposes into invariant subspaces of fixed spin carrying representations of local SU(2). The physical interpretation of this procedure is discussed.

I. Introduction

In the local current algebra formulation of non-relativistic quantum theory, systems of spinless particles are described by means of representations of the infinite-dimensional Lie algebra generated by the fixed-time operators $\varrho(f) = \int \varrho(\mathbf{x}) f(\mathbf{x}) d\mathbf{x}$ and $J(\mathbf{g}) = \int \mathbf{J}(\mathbf{x}) \cdot \mathbf{g}(\mathbf{x}) d\mathbf{x}$, where $\varrho(\mathbf{x})$ is the mass density, $\mathbf{J}(\mathbf{x})$ is the momentum density, and f and $\mathbf{g}$ are test functions vanishing at infinity. The corresponding infinite-dimensional Lie group is a semidirect product $\mathscr{S} \wedge \mathscr{K}$, where $\mathscr{S}$ is Schwartz' space of test functions under addition, and $\mathscr{K}$ is a group of diffeomorphisms of $\mathbb{R}^3$ under composition [1–3]. To describe non-relativistic particles with spin, however, it was thought necessary to introduce at the outset additional operators $\Sigma(\mathbf{h}) = \int \mathbf{\Sigma}(\mathbf{x}) \cdot \mathbf{h}(\mathbf{x}) d\mathbf{x}$, where $\mathbf{\Sigma}(\mathbf{x})$ is the spin density [4]. The Lie group generated by the Σ-operators is the local SU(2) current group [5–9].

We have seen in earlier work that a wide variety of distinct quantum-mechanical systems can be described by *unitarily inequivalent representations* of $\mathscr{S} \wedge \mathscr{K}$, without introducing additional physical quantities. For instance, representations of the diffeomorphism group describe fermions as well as bosons without the need for anticommuting field operators [10]. The question arises, then, whether one can obtain particle spin directly from irreducible repre-

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