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Convergence of $U(1)_3$ Lattice Gauge Theory to its Continuum Limit

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Abstract. It is shown that in three space-time dimensions the pure U(1) lattice gauge theory with Villain action and fixed coupling constant converges to the free electromagnetic field as the lattice spacing approaches zero. The same holds for the Wilson action on the electric sector.

1. Introduction

The lattice approximation to gauge field theories [22] beautifully preserves those differential geometric structures of the continuum theory which are important for physics. In addition to its value as a computational tool it provides a potential mechanism for proving the existence of continuum gauge theories.

Balian et al. [1] have shown that for pure gauge theories in two dimensions the compact lattice version is solvable, which yields, by explicit computation, a proof that the lattice theory indeed converges to a continuum limit in two dimensions. But in three or more dimensions compact lattice gauge theories are not solvable and no such model has heretofore been shown to have a continuum limit.

In three dimensions the compact pure gauge model with gauge group U(1) (= circle group) is expected to converge in the continuum limit to the free electromagnetic field. See for example [2]. In their deep paper on confinement in the $U(1)_3$ lattice gauge theory Göpfert and Mack [10] showed that the integer scalar field naturally associated to the dual model converges upon suitable normalization to a free scalar field if the coupling constant g is allowed to go to infinity at an appropriate rate as the lattice spacing goes to zero. They conjectured also that in the canonical limit in which g is held fixed the lattice gauge field itself should converge to the free electromagnetic field at the level of the field variables F_{uv} or the Wilson loops, at least for the Villain action.

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