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Exponential Lower Bounds to Solutions of the Schrödinger Equation: Lower Bounds for the Spherical Average

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Abstract. For a large class of generalized N-body-Schrödinger operators, H, we show that if $E<\Sigma=\inf\sigma_{\mathrm{ess}}(H)$ and ψ is an eigenfunction of H with eigenvalue E, then

$$\lim_{R\to\infty} R^{-1}\ln\left(\int_{S^{n-1}} |\psi(R\omega)|^2 d\omega\right)^{1/2} = -\alpha_0,$$

with α_0^2+E a threshold. Similar results are given for $E\geqq\Sigma$.

I. Introduction

In this paper we will be concerned with operators of the form

$$H=-\Delta+V(x) \tag{1.1}$$

in $L^2(\mathbb{R}^n)$, where

$$V(x)=\sum_{i=1}^{M} v_i(\pi_i x). \tag{1.2}$$

In (1.2) π_i is the orthogonal projection onto a subspace X_i of $\mathbb{R}^n$ and v_i is a real valued function on X_i satisfying

$$v_i(-\Delta_i+1)^{-1} \quad \text{is compact}, \tag{1.3}$$

$$(-\Delta_i+1)^{-1} y\cdot\nabla v_i(y)(-\Delta_i+1)^{-1} \quad \text{extends to a compact operator}. \tag{1.4}$$

Here Δ_i is the Laplacian in $L^2(X_i)$. By (1.4) we mean the following: Let $\mathscr{S}(X_i)$ be the Schwartz space of test functions on X_i and T_i the tempered distribution given by $y\cdot\nabla v_i(y)$. Define the sesquilinear form

$$q(f,g)=T_i((-\Delta_i+1)^{-1}\bar{f}(-\Delta_i+1)^{-1}g)$$

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