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Classical Limits for Quantum Particles in External Yang-Mills Potentials

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Abstract. We consider the limit $\hbar \to 0$ for nonrelativistic quantum particles moving in external Yang-Mills potentials. It is shown that the partition function and the solutions of the equations of motion converge to their corresponding classical counterparts.

1. Introduction

The equations governing the motion of a classical particle moving in an external SU(2) Yang-Mills potential were first obtained by Wong [Wo] by taking the formal limit $\hbar \to 0$ of the quantum mechanical equations of motion. Since then, various aspects of these equations have been studied and used [BSSW, BCL, BW, DC, Sch, S], as well as extended [GS, St] to arbitrary groups using the symplectic structure of coadjoint orbits [Ki, Ko, Sou]. We will review this classical formulation in Sect. 2 in a form suited for the applications we have in mind.

The principle of minimal coupling leads to a prescription of how to couple a quantum mechanical particle with internal degrees of freedom, like isospin in the SU(2) case, to an external Yang-Mills potential. In mathematical language, minimal coupling amounts to the replacement of the ordinary Laplace operator by the Laplace-Bochner operator obtained from the connection, whose Christoffel symbols just form the given Yang-Mills vector potential. This Laplace-Bochner operator thus describes the interaction with the "magnetic" part of the Yang-Mills potential and is of interest in quantum field theory, because in its euclidean formulation it describes the coupling of Higg's fields to Yang-Mills fields.

In addition the Hamiltonian may contain a scalar (with respect to space-time) Yang-Mills potential, describing an "electric" interaction. We present this setup in Sect. 3 together with some concepts from group theory needed in this context.

In the theory of quantum statistical spin systems it is well known that in order to obtain the corresponding classical theory when $\hbar \to 0$, it is necessary to let the

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