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Central Limit Theorem for the Lorentz Process via Perturbation Theory

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Abstract. The Markov partition of the Sinai billiard allows the following heuristic interpretation for the Lorentz process with a $\mathbb{Z}^2$-periodic configuration of scatterers: while executing a (non-Markovian) random walk on $\mathbb{Z}^2$, the particle changes its internal state according to the symbolic dynamics defined by the Markov partition. This picture can be formalized and then the Lorentz process appears as the limit of a sequence of (Markovian!) random walks with a finite but increasing number of internal states and the central limit theorem can be proved for it by perturbational expansions with uniformly bounded – in a sence related to the Perron-Frobenius theorem – coefficients and uniform remainder terms.

1. Introduction

In [K-Sz (1983)] the authors of the present paper proved a local central limit theorem for random walks with internal degrees of freedom (RWwIDF). These generalizations of the classical random walks had been introduced and studied by Sinai [S (1981)] in the hope they would help in understanding the Lorentz process. As a matter of fact, Gyires [Gy (1960)], in his studies on Toeplitz type hypermatrices, proved a local central limit theorem closely related to the theorem of [S (1981)]. His paper refers to a remark of Rényi, who also found a probabilistic interpretation of Gyires' result, namely just in terms of random walks with internal states (cf. also Gyires [Gy (1962)]). Our aim here is to justify Sinai's approach.

In fact, we give a new proof for the central limit theorem (CLT) obtained by Bunimovich and Sinai [B-S (1980)] for the Lorentz process with a periodic configuration of scatterers. At the price of having worked out a sort of uniform – for a family of matrices – perturbation theory, our arguments are simpler and they require less calculations. Moreover, we could immediately obtain more exact results, namely Chebyshev-Edgeworth-Cramér type asymptotic expansions in the

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