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# Axioms for Renormalization in Euclidean Quantum Field Theory

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**Abstract.** A set of axioms which fix Euclidean renormalizations up to a finite renormalization is proposed. There exists a one to one correspondence between Euclidean renormalizations and renormalizations in Minkowski space-time satisfying Hepp's axioms. No restrictions on masses are imposed.

## I. Introduction

In [6] Hepp proposed a general axiomatic framework for renormalization theory in perturbative relativistic quantum field theory. A renormalization is a map which assigns to unrenormalized Feynman amplitudes (which are not, in general, tempered distributions) the corresponding tempered distributions (called renormalized Feynman amplitudes) in such a way that some conditions are satisfied. These conditions are motivated by physical considerations, and they include: Lorentz covariance, unitarity and causality. It is shown that all renormalizations which satisfy the axioms are essentially equivalent: the only arbitrariness is a finite renormalization [6] (in renormalizable theories this arbitrariness is removed by fixing the values of masses and charges). Hepp showed also that the usual renormalization schemes satisfy his axioms.

In this paper we are concerned with renormalization theory in Euclidean space-time. The advantages of the Euclidean approach to field theory are well known: the matters simplify considerably. For the history and modern axiomatic results in this direction see [9]. In the context of renormalization theory this was recognized already by Dyson [1]. In order to avoid the problem of poles in the region of integration, he formally replaced $p^0$ by $ip^0$ in the $p$-space Feynman amplitudes. Weinberg [11] proved the celebrated power counting theorem giving sufficient conditions for the convergence of Euclidean amplitudes and describing their asymptotic behavior in $p$-space. Zimmermann [13] developed a $p$-space renormalization scheme, proved its convergence, and related it to relativistic Feynman amplitudes. In the context of one-loop approximation to the $\phi^4$ theory, renormalized Euclidean amplitudes were also discussed by Williams [12].