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On an Elaboration of M. Kac's Theorem Concerning Eigenvalues of the Laplacian in a Region with Randomly Distributed Small Obstacles

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Abstract. We remove m-balls of centers $w_1,\ldots,w_m$ with the same radius α/m from a bounded domain Ω in $\mathbf{R}^3$ with smooth boundary γ. Let $\mu_k(\alpha/m;w(m))$ denote the k-th eigenvalue of the Laplacian in $\overline{\Omega\backslash m\text{-balls}}$ under the Dirichlet condition. We consider $\mu_k(\alpha/m;w(m))$ as a random variable on a probability space $(w_1,\ldots,w_m)\in\Omega\times\cdots\times\Omega$ and we examine a precise behaviour of $\mu_k(\alpha/m;w(m))$ as $m\to\infty$. We give an elaboration of. M. Kac's theorem.

1. Introduction

We consider a bounded domain Ω in $\mathbf{R}^3$ with smooth boundary γ. Let $B(\varepsilon;w)$ be the ball defined by $B(\varepsilon;w)=\{x\in\mathbf{R}^3;|x-w|<\varepsilon\}$. Let $0<\mu_1(\varepsilon;w(m))\leqq\mu_2(\varepsilon;w(m))\leqq\mu_3(\varepsilon;w(m))<\cdots$ be the eigenvalues of $-\Delta(=-\text{div grad})$ in $\Omega_{\varepsilon,w(m)}=\Omega\backslash\bigcup_{i=1}^{m}B(\varepsilon;w_i^{(m)})$ under the Dirichlet condition on its boundary. Here $w(m)$ denotes the set of m-points $\{w_1^{(m)}\}_{i=1}^m$. We arrange $\mu_k(\varepsilon;w(m))$ repeatedly according to their multiplicities.

Let $V(x)\geqq 0$ be a C^1 function on $\bar{\Omega}$ satisfying

$$\int_\Omega V(x)\,dx=1.$$

Then, we consider Ω as the probability space with the probability $V(x)\,dx$. Let $\Omega^m=\prod_{i=1}^m\Omega$ be the probability space with the product measure.

The aim of this note is to prove the following:

Theorem 1. *Fix $\alpha>0$ and k. Then,*

$$\lim_{m\to\infty}\mathbb{P}(w(m)\in\Omega^m;m^{\tilde{\delta}}|\mu_k(\alpha/m;w(m))-\mu_k^V|<\varepsilon)=1 \tag{1.1}$$

for any $\varepsilon>0$ and $\tilde{\delta}\in[0,\frac{1}{4})$. Here μ_k^V *denotes the k^{th} eigenvalue of $-\Delta+4\pi\alpha V(x)$ in Ω under the Dirichlet condition on γ.*