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Scaling Limit of Some Critical Models

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Abstract. In this paper we consider massless systems which are strong perturbations of the massless lattice free field. Under quite general assumptions on the potential, we prove that the continuum (scaling) limit of these systems is Gaussian.

1. Introduction

In this paper we study the (continuum) scaling limit of some massless models of classical statistical mechanics. The main particularity of the systems considered is that their correlations are not absolutely summable; typically they have a clustering like $|x|^{-d}$ in d dimensions. Such a situation arises for the low temperature rotator model [1] or for the lattice dipole gas [2]. These systems are rather well approximated by the following model of an anharmonic crystal, defined on $\mathbb{Z}^d$ and described by the Hamiltonian [3]:

$$H=\tfrac{1}{2}\sum_x (\nabla_x\phi)^2+\lambda/4\sum_x(\nabla_x\phi)^4, \tag{1}$$

where ϕ_x is a real random variable uniformly distributed on the real line. For small coupling λ the (block spin) scaling limit of this model can be obtained using the machinery of Gawedzki and Kupiainen based on rigorous renormalization group arguments as announced in [4] (see also [5] for related results by Magnen and Sénéor). For large coupling λ, the question of the scaling limit is, so far, totally open.

In [6] we developed a method essentially based on correlation inequalities to obtain bounds (uniform in λ) on the long distance behaviour of general correlation functions of model (1). We feel that those bounds should be useful to study the scaling limit of this model for arbitrary λ.

In this paper we consider a simplification of model (1): the Hamiltonian that we choose is

$$H=\tfrac{1}{2}\sum_x (\nabla_x\phi)^2+\lambda/4\sum_x((-\Delta)^\alpha\phi_x)^4, \tag{2}$$

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