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Low Temperature Expansions for the Gibbs States of Weakly Interacting Quantum Ising Lattice Systems

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Abstract. Low temperature expansions for the Gibbs states of weakly interacting transverse Ising-like models are developed, by conditioning the states on a sub-algebra of observables. The conditioned states have effective classical Hamiltonians which are estimated by the solution to a simple implicit equation. Provided the interaction is sufficiently weak but fixed independent of temperature, and the temperature is sufficiently low, exponential clustering of the correlation functions holds.

1. Introduction

Let $H_\Lambda(\varepsilon)$ be the transverse Ising Hamiltonian associated with a finite volume $\Lambda \subset \mathbb{Z}^\nu$,

$$H_\Lambda(\varepsilon) = -\sum_{i\in\Lambda} \sigma^x(i) - \varepsilon \sum_{A\subset\Lambda} \hat{V}_\Lambda(A)\sigma^z(A). \tag{1.1}$$

Here, $\sigma^z(A) = \bigotimes_{i\in A} \sigma^z(i)$ and $\sigma^x(i)$ and $\sigma^z(i)$ are the Pauli spin matrices acting at the site i

$$\sigma^x(i) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^z(i) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{1.2}$$

taken in a basis so the first term of the Hamiltonian is a spin-flip term, the second a classical Ising term. To simplify the analysis, we assume that $H_\Lambda(\varepsilon)$ is translation invariant, i.e. that Λ is rectangular and that periodic boundary conditions are imposed. Finally, we assume that the coefficients $\hat{V}_\Lambda(A)$, which are real, are equal to zero for the cardinality of A, $|A|$, exceeding some constant C, independent of Λ.

The purpose of this article is to show if ε is fixed and sufficiently small, then the Gibbs state corresponding to $H_\Lambda(\varepsilon)$,

$$\langle X \rangle_{\Lambda,\varepsilon,\beta} = (\operatorname{tr}\exp(-\beta H_\Lambda(\varepsilon)))^{-1} \operatorname{tr} X \exp(-\beta H_\Lambda(\varepsilon)) \tag{1.3}$$

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