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Lattice Yang-Mills Theory at Nonzero Temperature and the Confinement Problem

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Abstract. We discuss finite temperature lattice Yang-Mills theory with special attention to the confinement problem. The relationship between the confinement criteria of Wilson, Polyakov, and 't Hooft is clarified by establishing a string of inequalities between the corresponding string tensions.

The close connection between finite temperature Yang-Mills models and spin models is exploited to obtain new and rather sharp upper bounds for the critical coupling constant above which there is confinement. This same analogy also allows us to establish infrared bounds for the gauge models that yield a lower bound for this critical coupling and thereby show the existence of a weak coupling regime without confinement at nonzero temperature in three or more space dimensions.

Finally we discuss extension of our results to other forms of the lattice action, the Hamiltonian lattice models of Kogut and Susskind and 't Hooft's $N \to \infty$ limit.

I. Introduction

One of the outstanding problems of quantum field theory is to understand the so-called confinement of quarks. In the most general sense this means the question why the particle content of quantum chromodynamics (QCD) consists only of hadrons but not of anything like quarks or gluons (not even in bleached form). Since this is an almost intractable problem, at least for the moment, one normally considers some simplified version of it.

First of all, since (continuum) QCD has not been constructed so far, one uses the device of replacing space-time or at least space by a lattice; together with some more or less well-founded scaling hypotheses this still allows one to gain insight into the properties of the continuum theory. On the other hand the well-developed machinery of lattice statistical mechanics becomes available for the analysis.

The next drastic step that is conventionally employed consists of eliminating the quarks from the theory and only considering "quark test charges" or "infinitely