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Unitary Equivalence of Temperature Dynamics for Ideal and Locally Perturbed Fermi-Gas

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Abstract. We consider the local perturbation

$$V=\varepsilon \sum_{x,\, y\in\mathbb{Z}^\nu} V(x,y)\chi_\Omega(x)\chi_\Omega(y)a^*(x)a^*(y)a(y)a(x)$$

of the ideal Fermi-gas on the lattice $\mathbb{Z}^\nu$, where Ω is a finite subset of $\mathbb{Z}^\nu$ and χ_Ω is its indicator. The invertibility of Möller morphisms for small ε is proven. It follows that in the cyclic GNS representation with respect to KMS states the dynamics of ideal and locally perturbed Fermi-gas are unitary equivalent.

Introduction

Two kinds of equilibrium states are usually considered in mathematical physics: ground (zero temperature) and KMS (nonzero temperature) states. There are many results concerning spectral decompositions of Hamiltonians (in the GNS-representation) for the ground state representations which support the so-called quasi-particle picture: any system is a collection of noninteracting quasiparticles (we note that asymptotic completeness is not proven even for the ground state representations).

In Appendix B we explicitly calculate spectral decomposition of H_0 showing the quasiparticle picture. We could not find this representation in the literature.

For the KMS-states the only results in this direction are due to Robinson, Evans etc. [1–3], who proved the existence of Möller morphisms for local perturbations of quasi-free systems. In this paper we prove the invertibility of these morphisms. An extension of our method and other results will appear in subsequent publications.

1. Formulation of the Main Result

Let $K=l_2(\mathbb{Z}^\nu)$ be a complex Hilbert space and $\mathfrak{A}=\mathfrak{A}(K)$ be the CAR-algebra over K. It is well known that its generators $a(f)$, $a^*(f)$, $f\in K$ satisfy the following