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Graded Gauge Theory

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Abstract. The mathematical background for a graded extension of gauge theories is investigated. After discussing the general properties of graded Lie algebras and what may serve as a model for a graded Lie group, the graded fiber bundle is constructed. Its basis manifold is supposed to be the so-called superspace, i.e. the product of the Minkowskian space-time with the Grassmann algebra spanned by the anticommuting Lorentz spinors; the vertical subspaces tangent to the fibers are isomorphic with the graded extension of the SU(N) Lie algebra. The connection and curvature are defined then on this bundle; the two different gradings are either independent of each other, or may be unified in one common grading, which is equivalent to the choice of the spin-statistics dependence. The Yang-Mills lagrangian is investigated in the simplified case. The conformal symmetry breaking is discussed, as well as some other physical consequences of the model.

1. Construction of a Graded Lie Algebra Associated with a Lie Group G

Let G be a Lie group of dimension N; in what follows, it will be supposed compact and semi-simple, unless explicitly stated otherwise. Let $\mathscr{A}_G$ denote its Lie algebra; for $X, Y \in \mathscr{A}_G$ their skew product is $[X, Y]$ and satisfies

$$[X, Y] = -[Y, X], \tag{1.1}$$

and the Jacobi identity

$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0. \tag{1.2}$$

The adjoint representation of $\mathscr{A}_G$ is defined as the mapping

$$\mathrm{ad} : \mathscr{A}_G \to L(\mathscr{A}_G, \mathscr{A}_G), \tag{1.3}$$

such that

$$\mathrm{ad}(X)\, Y = [X, Y]\,; \tag{1.4}$$