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A General Central Limit Theorem for FKG Systems*

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Abstract. A central limit theorem is given which is applicable to (not necessarily monotonic) functions of random variables satisfying the FKG inequalities. One consequence is convergence of the block spin scaling limit for the magnetization and energy densities (jointly) to the infinite temperature fixed point of independent Gaussian blocks for a large class of Ising ferromagnets whenever the susceptibility is finite. Another consequence is a central limit theorem for the density of the *surface* of the infinite cluster in percolation models.

1. Introduction

For a translation invariant d-dimensional system of L_2 random variables (or random vectors), $\{X_k : k \in \mathbb{Z}^d\}$, we define for each $n = 1, 2, \ldots$, the block variables

$$X_k^n = n^{-d/2} \sum_{j \in B_k^n} (X_j - EX_j),$$

where B_k^n is a block of side length n located near nk,

$$B_k^n = \{j : nk_l \leqq j_l < n(k_l + 1) \text{ for } l = 1, \ldots, d\} = nk + B_0^n,$$

and E denotes expectation. In [N1] a central limit theorem for $\{X_k^n : k \in \mathbb{Z}^d\}$ as $n \to \infty$ was obtained under the additional assumptions that the X_k's obey the FKG inequalities [FKG] and $\sum_k \mathrm{Cov}(X_0, X_k) < \infty$.

In the context of a general Ising model, $\{\sigma_k : k \in \mathbb{Z}^d\}$ with energy density,

$$\mathscr{E}_k = -\sum_{j \in \mathbb{Z}^d} J(j-k)\sigma_j\sigma_k - h\sigma_k \quad \left(J(j) \geqq 0 \ \forall j \text{ and } \sum_{j \in \mathbb{Z}^d} J(j) < \infty\right),$$

and single site distribution $d\varrho(\sigma_j)$, the central limit theorem of [N1] implies convergence of the σ_j^n's to independent mean zero normal random variables of

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