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The Existence of a Black Hole Due to Condensation of Matter

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Abstract. When enough matter is condensed in a small region, gravitational effects will be strong enough to cause collapse and a black hole will be formed. We formulate and prove here such a statement in the language of general relativity. (This is Theorem 2 of this paper.)

The main result of this paper is that for an asymptotically flat initial data set, with the mass density large on a large region, there is an apparent horizon (and a closed trapped surface) in the initial data. It was shown by Penrose [2] and Hawking [1] that under physically reasonable assumptions, the existence of a closed trapped surface implies that the spacetime which evolves from the initial data contains a black hole. Simple examples show that the mass density can be large on a set of arbitrarily large *diameter* without the existence of an apparent horizon in the initial data. Therefore, the notion of a "large region" must be suitably defined. We formulate a notion which measures more than one direction in Ω.

Definition. Let Γ be a simple closed curve in Ω which bounds a disk in Ω. We let $N_r(\Gamma)$ denote the set of points within a distance r of Γ. Define the *H-radius of* Ω *with respect to* Γ by

$$\operatorname{Rad}(\Omega,\Gamma)=\sup\{r:\operatorname{dist}(\Gamma,\partial\Omega)>r,\ \Gamma \text{ does not bound a disk in } N_r(\Gamma)\}.$$

We define the *H-radius of* Ω, denoted $\operatorname{Rad}(\Omega)$, by

$$\operatorname{Rad}(\Omega)=\sup\{\operatorname{Rad}(\Omega,\Gamma):\Gamma \text{ as above}\}.$$

Remark 1. Note that if Ω were a ball of radius R in $\mathbb{R}^3$, then $\operatorname{Rad}(\Omega)=R/2$; on the other hand if Ω is the cross product of $S^2(R)$ with an interval $(-L,L)$, then $\operatorname{Rad}(\Omega)=\min\left\{\frac{\pi R}{2},L\right\}$.