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Counterexamples to Some Results on the Existence of Field Copies*

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Abstract. Several criteria are known for determining which connections A are determined uniquely by their curvature F, or by F and its covariant derivatives. On a principal bundle with semi-simple gauge group G over a 4-manifold M, a sufficient condition for F to determine A uniquely is that the linear map $B \to [F \wedge B]$ from Lie algebra-valued 1-forms to 3-forms (pulled back to M via a local gauge) be invertible on an open dense set in M. Recently F. A. Doria has claimed that this condition is also necessary. We present counterexamples to this claim, and also to his assertion that F determines A uniquely if the restriction of the bundle to every open subset of M has holonomy group equal to G and F is "not degenerate as a 2-form over spacetime."

1. Background and Results

In recent years there has been an interest in finding criteria for the existence of field copies, that is, of gauge fields (= field strengths = curvatures of a principal bundle) which arise from two or more gauge potentials (connections) which are not gauge equivalent (see, for example, [C, DX, DD, DW, GY, H1, H2, KC, M, R, So, W, and WY]). This problem is closely related to questions of when a connection A is determined uniquely by its curvature F, or by F together with other data (in the absolute sense, that is, ignoring gauge equivalence). Among the numerous results on these questions are two types of general criteria, which we shall call algebraic and geometric.

The algebraic criterion ([C, DT, DW, H1, MS, R]), applicable when the base space M is a 4-dimensional manifold, is based on the Bianchi identity. Specialized to the trivial principal G-bundle $\mathbb{R}^4 \times G$ over $M = \mathbb{R}^4$, it says that a sufficient condition for F to determine A uniquely is that the linear transformation $\mathrm{ad}_F: B \to [F \wedge B]$ from $\mathfrak{g}$-valued 1-forms to $\mathfrak{g}$-valued 3-forms on M ($\mathfrak{g}$ = Lie algebra of G) be invertible at all x in an open dense subset of M. (See [MS] for the generalization of this, in

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