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Classical Equations $dN_i/dr = \frac{1}{2} i\varepsilon_{ijk}[N_j, N_k]$

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Abstract. We study the first order system of equations $dN_i/dr = \frac{1}{2} i\varepsilon_{ijk}[N_j, N_k]$, where the N_i are classical, "non-abelian" gauge-Higgs fields with spherical symmetry. Exact solutions are constructed.

1.

Our starting point are the Bogomolny equations (vanishing self-coupling for the adjoint Higgs field), when there is spherical symmetry

$$\frac{d}{dr}\psi = \tfrac{1}{2}[N^+, N^-], \quad \frac{d}{dr}N^\pm = \pm[\psi, N^\pm], \tag{1}$$

$$[T_3, \psi] = 0, \qquad [T_3, N^\pm] = \pm N^\pm. \tag{2}$$

Here T_3 is a generator of the SO(3) subgroup of our gauge group G, and $\psi, N^\pm$ (related to the original Higgs and gauge fields) are elements of the Lie algebra $L(G)$ and satisfy $\psi = \psi^+$, $(N^+)^+ = N^-$ (Hermitean conjugates). The connection between these variables and the title variables N_i is $N_3 = -\psi$ and $N^\pm = N_i \pm iN_2$. For a derivation of the above equations see [1]. We also use the notion of the "grade" n of a generator X, if

$$[T_3, X] = nX.$$

The grade n is an eigenvalue of T_3, and therefore an integer or half-integer. We also need the star (*) operation

$$X^* = (-1)^{T_3} X^+ (-1)^{T_3},$$

which defines an involution of the subalgebra of $L(G)$ with integer grades.

Define $R = -\psi + N^-$ and $R^* = -\psi - N^+$. Using (1) one finds that

$$\frac{d}{dr}(R + R^*) = [R^*, R]. \tag{3}$$

The reverse is also true: given a Lie algebra element R which consists of a grade

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