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Infrared and Ultraviolet Dimensional Meromorphy of Feynman Amplitudes

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Abstract. By the concurrent use of dimensional and analytic regularizations with the complete Mellin (CM) representation, we find in a direct way the ultraviolet and infrared poles in space-time dimension, for any Feynman amplitude with an arbitrary subset of vanishing masses.

I. Introduction

A dimensionally regularized Feynman amplitude [1] is the analytic continuation, in the space-time dimension D, of the function $A_G(D)$ defined by the Feynman integral corresponding to a given graph G. When there is no vanishing internal mass, it is well known that the integral exists for a sufficiently low value of Re D, and defines a meromorphic function of D: the singularities of $A_G(D)$ are poles, located at real rational values of D. We denote by D_{UV} the first pole, that is the lowest value of D for which the Feynman integral presents ultraviolet (UV) divergences.

Now if there are vanishing masses, the Feynman integral may present infrared (IR) divergences for Re $D \leqq D_{\mathrm{IR}}$. When all masses vanish, it has been shown that $A_G(D)$ remains meromorphic, with new "infrared" poles [2]. But in the literature there seems to be no such result for the Feynman integrals with only a partial subset of vanishing masses: here we prove the meromorphy of $A_G(D)$ in this more general situation.

In Sect. II we look at the case $D_{\mathrm{IR}} < D_{\mathrm{UV}}$. Then $A_G(D)$ is defined by the Feynman integral for $D_{\mathrm{IR}} < \mathrm{Re}\, D < D_{\mathrm{UV}}$. And we use the CM representation [3] to prove that the singularities for $\mathrm{Re}\, D \leqq D_{\mathrm{IR}}$ are still poles, located at real rational values of D.

In the case $D_{\mathrm{IR}} \geqq D_{\mathrm{UV}}$, the formal Feynman integral exists nowhere. In Sect. III, we use the analytic regularization [4] to define $A_G(D)$, and we extend the results of Sect. II to this case.

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